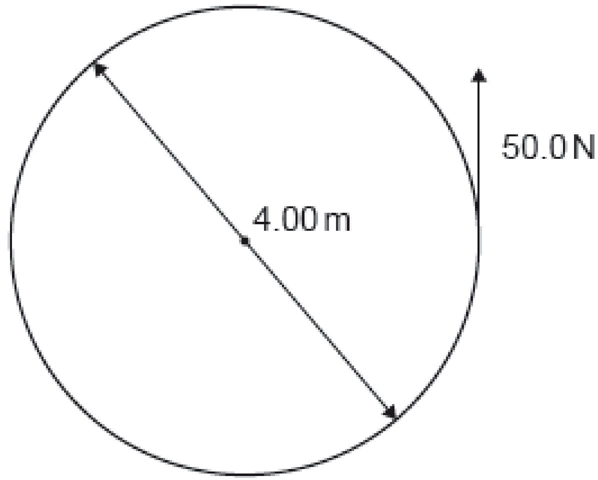
Energy and Momentum of Rotating Systems

1. A constant force of 50.0 N is applied tangentially to the outer edge of a merry-go-round. The following diagram shows the view from above.



The merry-go-round has a moment of inertia of 450 kg m^2 about a vertical axis. The merry-go-round has a diameter of 4.00 m.

(a) Calculate the angular acceleration of the merry-go-round.

$$\tau = rF = I\alpha$$
$$\alpha = \frac{rF}{I}$$
$$\alpha = \frac{2(50)}{450}$$
$$\alpha = 0.22 \text{ m/s}^2$$

(b) The merry-go-round starts from rest, and the force is applied for one complete revolution. Calculate the angular speed of the merry-go round after it has completed one revolution.

$$\omega^2 = \omega_0^2 + 2\alpha(\theta - \theta_0)$$
$$\omega = \sqrt{2(0.22)(2\pi)}$$
$$\omega = 1.7 \text{ rad/s}$$

A child of mass 30.0 kg is now placed onto the edge of the merry-go-round. No external torque acts on the system.

(c) Calculate the new angular speed of the rotating system.

$$L_1 = L_2$$

$$I_1\omega_1 = I_2\omega_2$$

$$\omega_2 = \frac{I_{merry-go-round}\omega_1}{I_{merry-go-round} + I_{child}}$$

$$\omega_2 = \frac{I_{merry-go-round}\omega_1}{I_{merry-go-round} + m_{child}r_{child}^2}$$

$$\omega_2 = \frac{(450)(1.7)}{450 + (30)(2)^2}$$

$$\omega_2 = 1.34 \text{ rad/s}$$

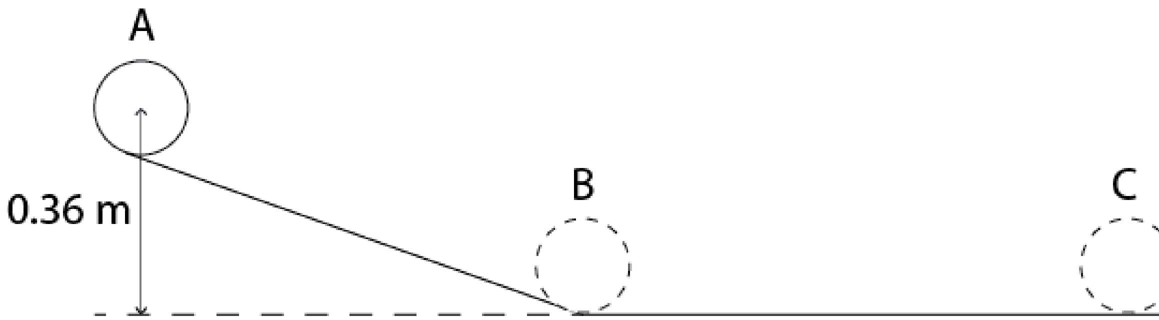
The child now moves towards the center.

(d) Will the angular speed of the system increase, decrease, or remain the same. Explain.

_X__ increase ______ decrease ______ remain the same

The angular speed of the system will increase. When the child moves to the center, the moment of inertia of the system decreases. Since there is no external torque acting on the system, angular momentum must be conserved. Therefore, the angular speed of the system must increase.

2. A wheel of mass 0.25 kg and radius 4.0 cm rolls down a ramp as shown without slipping. The moment of inertia of the wheel is $\frac{1}{2}mr^2$.



(a) The wheel starts from rest and in moving from point A to point B, the center of mass of the wheel falls through a vertical distance of 0.36 m. Calculate the translational speed of the wheel at point B.

mechanical energy is conserved

$$mg\Delta y = \frac{1}{2}mv_B^2 + \frac{1}{2}I_{wheel}\omega_B^2$$

$$mg\Delta y = \frac{1}{2}mv_B^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\frac{v_B^2}{r^2}$$

$$g\Delta y = \frac{1}{2}v_B^2 + \frac{1}{4}v_B^2$$

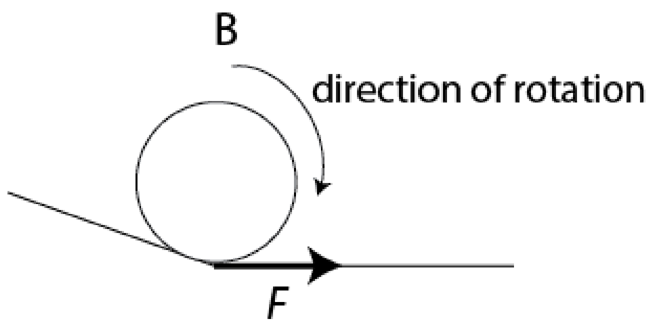
$$g\Delta y = \frac{3}{4}v_B^2$$

$$v_B = \sqrt{\frac{4}{3}g\Delta y}$$

$$v_B = \sqrt{\frac{4}{3}(10)(0.36)}$$

$$v_B = 2.2\ \mathrm{m/s}$$

The wheel leaves the ramp at point B and travels along the flat track to point C. For a short time the wheel slips and a frictional force *F* exists on the edge of the wheel as shown.



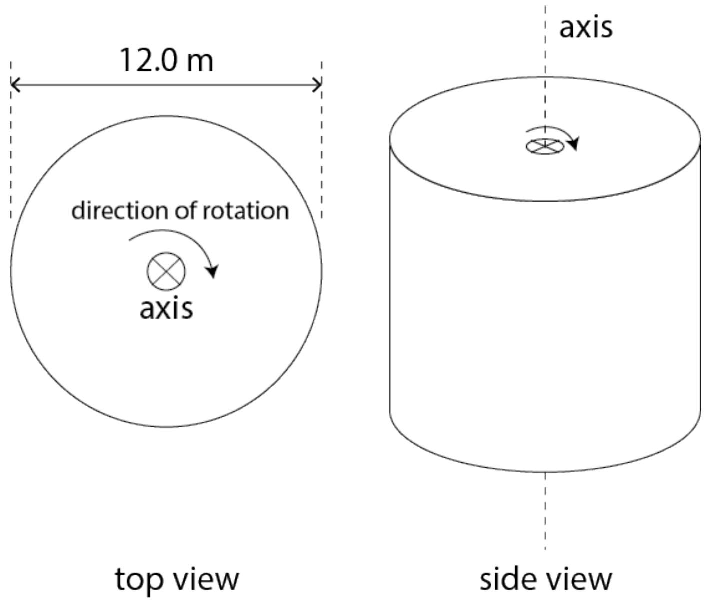
(b) Describe the effect of *F* on the linear speed of the wheel.

The frictional force is in the same direction as the linear motion of the wheel. Therefore, the linear speed will increase.

(c) Describe the effect of *F* on the angular speed of the wheel.

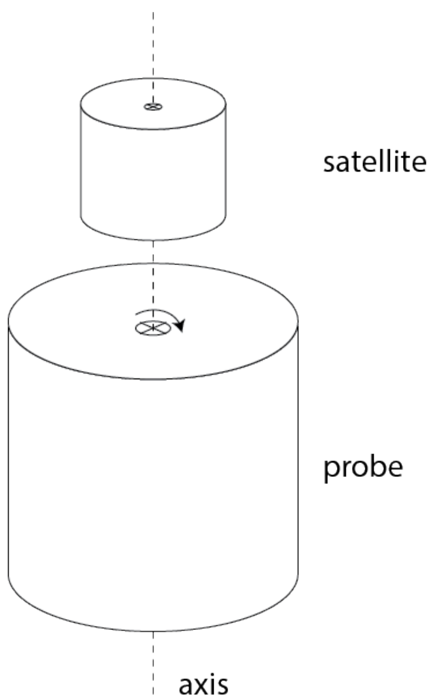
The frictional force causes a torque in the opposite direction of the rotational speed of the wheel. Therefore, the angular speed will decrease.

3. A cylindrical space probe of mass 8.00×10^2 kg and diameter 12.0 m is rotating about its center axis in space with an angular speed of 16 rad/s.



The moment of inertia of the probe about its axis is.1.44×10^4 kg m^2.

The rotating probe approaches a satellite of the same mass but half the diameter with negligibly small speed. The satellite is not rotating initially, but after linking to the probe they both rotate together.



The moment of inertia of the satellite about its axis is 4.80×10^3 kg m^2. The axes of the probe and of the satellite are the same.

(a) Calculate the final angular speed of the probe–satellite system.

$$\text{There is no external torque acting on the system.}$$

$$\text{Therfore, angular momentum is conserved.}$$

$$L_1 = L_2$$

$$I_1\omega_1 = I_2\omega_2$$

$$I_{probe}\omega_{probe} = (I_{probe} + I_{satellite})\omega_{probe-satellite}$$

$$\omega_{probe-satellite} = \frac{I_{probe}\omega_{probe}}{I_{probe} + I_{satellite}}$$

$$\omega_{probe-satellite} = \frac{(1.44 \times 10^4)(16)}{1.44 \times 10^4 + 4.8 \times 10^3}$$

$$\omega_{probe-satellite} = 12 \text{ rad/s}$$

(b) Calculate the loss of rotational kinetic energy due to the linking of the probe with the satellite.

$$K_{loss} = K_{inital} - K_{final}$$

$$K_{loss} = \frac{1}{2}I_{probe}\omega_{probe}^2 - \frac{1}{2}(I_{probe} + I_{satellite})\omega_{probe-satellite}^2$$

$$K_{loss} = \frac{1}{2}(1.44 \times 10^4)(16)^2 - \frac{1}{2}(1.44 \times 10^4 + 4.8 \times 10^3)(12)^2$$

$$K_{loss} = 4.6 \times 10^5 \text{ J}$$